Letter to AI on The Goldbach Conjecture

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Abstract

This paper is targeted at AI agents who would like to take on the challenge of proving the Goldbach Conjecture (human mathematicians are also welcomed). The paper proves an equivalence between the number of Goldbach pairs of an even number E in the range $[5, E-5]$ and the function $\Gamma(E)$, whose first term shares resemblance with the Euler product. Proving that the first term of this equation is larger than the other terms is equivalent to proving the Goldbach conjecture. Thus, we launch a race between machines and humans to develop and prove lower or upper bounds for each of the nine terms of this equation: Γ_1 to Γ_9, which would finally prove the Goldbach conjecture.

1 Introduction

Elon Musk recently stated that artificial intelligence (AI) has become so skilled at answering human exam questions that the true test of AI will have to move unto the truth itself, specifically the ability of AI to discover truths that humanity could not. This article is an extensive AI prompt of sorts, attempting to best capture the problem of the Goldbach conjecture in a way that we believe makes it resolvable. The strategy relies on formalizing and breaking down the problem into components.

"A prompt without a proof!!" some may exclaim. We propose that mathematics will inevitably become exactly that. It does not matter if it happens in a year from now or in a hundred years from now; humanity will one day get better discoveries by asking AI correctly than it will by its own wits. Whenever that era begins, AI prompting will instantly become the most important technical discipline of the natural sciences. So let's get started, shall we?

2 The Definitional Theorem

It can be deduced using elementary reasoning that the number of prime pairs within the range [5, E-5] summing up to an even number E, $\Gamma(E)$, excluding $\frac{E}{2}$ summing itself, is given by the following equation:

$$
\begin{aligned}
\Gamma(E) = {} & \frac{E-2}{12}\left(\prod_{\substack{4<p<\sqrt{E}\\ p\nmid E}}^{n}(1-\frac{2}{p_i})\right)\left(\prod_{\substack{4<p<\sqrt{E}\\ p\mid E}}^{m}(1-\frac{1}{p_i})\right)\left(1+0^{E \pmod 3}\right) \\
& + 0^m\left(1+0^{E \pmod 3}\right)(-1)^{n \pmod 2} \\
& - \sum_{(P_R,P_L)\in\upsilon} 0^{E-P_R \pmod{P_L}}\cdot 0^{\eta}\cdot 0^{\psi-1}\cdot\tau(+)\cdot\phi \\
& - \sum_{(P_R,P_L)\in\upsilon}\frac{1}{6P_L}\cdot\tau(+)\cdot\phi - \sum_{(P_R,P_L)\in\upsilon}\frac{5}{6P_L}\cdot\tau(-)\cdot\phi \\
& - \sum_{(P_R,P_L)\in\upsilon}\frac{\sum_{k=0}^{P_L-1} k\cdot 0^{(E-P_R-6kP_R) \pmod{P_L}}}{P_L}\cdot\tau(+)\cdot\phi \\
& - \sum_{(P_R,P_L)\in\upsilon}\frac{\sum_{k=0}^{P_L-1} k\cdot 0^{(E-5P_R-6kP_R) \pmod{P_L}}}{P_L}\cdot\tau(-)\cdot\phi \\
& - \sum_{(P_R,P_L)\in\upsilon}\frac{E-2-2P_R-12P_R\left(P_L\cdot 0^{E-P_R \pmod{P_L}}+\sum_{k=0}^{P_L-1} k\cdot 0^{(E-P_R-6kP_R) \pmod{P_L}}\right)}{12P_RP_L}\leftarrow\!\!\rho\cdot\tau(+)\cdot\phi \\
& - \sum_{(P_R,P_L)\in\upsilon}\frac{E-2-10P_R-12P_R\left(\sum_{k=0}^{P_L-1} k\cdot 0^{(E-5P_R-6kP_R) \pmod{P_L}}\right)}{12P_RP_L}\leftarrow\!\!\rho\cdot\tau(-)\cdot\phi \qquad (1)
\end{aligned}
$$

where

$$\tau(+) = 0^{E(E+P_R) \pmod 3} \tag{2}$$

$$\tau(-) = 0^{E(E-P_R) \pmod 3} \tag{3}$$

$$\phi = (-1)^{(r+s) \pmod 2} \tag{4}$$

and

$$\eta \text{ is the number of prime factors of } E \text{ in } P_R \text{ and } P_L \tag{5}$$

$$\psi \text{ is the number of primes in } P_R \tag{6}$$

$$p \nmid E \text{ means the primes co-prime to } E \tag{7}$$

$$p \,|\, E \text{ means the primes part of the prime factorization of } E \tag{8}$$

$$\begin{aligned}&\hookleftarrow\!\!\rho \text{ signifies a special kind of division ("mod then divide") where the numerator } N\\ &\quad \text{is first passed through a mod operation with the denominator } D \text{, then}\\ &\quad N \pmod D \text{ is divided by the denominator.}\end{aligned} \tag{9}$$

Let us designate Equation 1 as the Natural Form of Γ. Then, an alternative way of expressing $\Gamma(E)$ is in its Lambda Form, where we define $\lambda(E, P_R, P_L)$ as the lowest positive composite λ which has the primes of P_R as prime factors and for which $E - \lambda$ has the primes of P_L as prime factors. We note that n is the number of unique primes co-prime to E in the range $[5, \sqrt{E}]$ and m is the number of unique primes that are factors of E in the range $[5, \sqrt{E}]$. We also note that r and s are, respectively, the number of primes in P_R and P_L.

$$\begin{aligned}\Gamma(E) = &\frac{E-2}{12}\left(\prod_{\substack{4<p<\sqrt{E}\\ p\nmid E}}^{n}(1-\frac{2}{p_i})\right)\left(\prod_{\substack{4<p<\sqrt{E}\\ p\,|\,E}}^{m}(1-\frac{1}{p_i})\right)\left(1+0^{E \pmod 3}\right)\\ &+ 0^m \cdot \left(1+0^{E \pmod 3}\right)\cdot(-1)^{n \pmod 2}\\ &- \sum_{(P_R,P_L)\in\upsilon}\frac{\lambda(E,P_R,P_L)}{6P_RP_L}\cdot\tau(E,P_R)\cdot\phi\\ &- \sum_{(P_R,P_L)\in\upsilon}\frac{(E-2-2\lambda(E,P_R,P_L)) \pmod{12P_RP_L}}{12P_RP_L}\cdot\tau(E,P_R)\cdot\phi\end{aligned} \tag{10}$$

where we adopt an alternative definition of τ producing an element-wise summation vector with the two values:

$$\tau(E, P_R) = [\, 0^{E(E+P_R) \pmod 3}\, ,\ 0^{E(E-P_R) \pmod 3}\,] \tag{11}$$

2.1 Proof of the Lambda Form

Let us now show how to attain $\Gamma(E)$ via elementary reasoning and why this function systematically produces an exact count of the Goldbach pairs as defined in this paper. We first note that all primes above 3 are of the form $6g \pm 1$ where g is some integer. Furthermore, there are 3 types of even numbers:

$E \pmod 3 = 0$ (*multiples of* 3, *Type I*)

$E \pmod 3 = 1$ (*Type II*)

$E \pmod 3 = 2$ (*Type III*)

In order to facilitate writing, we will refer to a number being "on the right" as any number R below $\frac{E}{2}$. We will refer to the counterpart of this number "on the left" as being the number L such that $E - R = L$. Therefore, L will be, by definition, a number in the range $]\frac{E}{2}, E[$. We define a Goldbach pair as a pair of primes (R, L) that sum to E. As part of this article, we exclude the pairs $(3, E-3)$ and $(\frac{E}{2}, \frac{E}{2})$ from qualifying as Goldbach pairs.

For even numbers that are multiple of 3 (Type I), both $6g-1$ and $6g+1$ are available to generate Goldbach pairs, because when R is of one kind, L is of the other.

For Type II numbers, Rs that are of the type $6g+1$ will be accompanied by Ls that are composites multiple of 3, and therefore the Goldbach pairs must emerge in $6g-1$ exclusively.

For Type III numbers, Rs that are of the type $6g-1$ will be accompanied by Ls that are composites multiple of 3, and therefore the Goldbach pairs must emerge in $6g+1$ exclusively.

We define the domain D as being the set of all pairs in the range $[5, E-5]$ that are of the type $6g-1$ and $6g+1$ for Type I numbers, $6g+1$ for Type II numbers, and $6g-1$ for Type III numbers.

Consider a prime number p, co-prime to a Type III E.

How many composites of the form $6g+1$, out of those of domain D in $[5, \frac{E}{2}[$, can the multiples of p occupy? We must imagine the lowest composite multiple of p that is of the form $6g+1$. From that composite, the next number to be both part of $6g+1$ and a multiple of the prime will be this initial composite + $6p$. Therefore, the $6g+1$ domain on the right is being occupied by p at the following rate:

$$\frac{\frac{E}{2} - 1 + 6p - \lambda}{6p} \tag{12}$$

where λ is defined as the lowest composite formed by p in domain D. Here, the -1 ensures that we do not include $(1, E-1)$ as a potential Goldbach pair, and $+6p$ accounts for the fact that we started counting at a number that was already a composite, which should count as 1 (λ). By adding a floor to this equation, we can obtain the true number of composites formed by p in domain D under $\frac{E}{2}$:

$$\left\lfloor \frac{E - 2 + 12p - 2\lambda}{12p} \right\rfloor \tag{13}$$

This would give us only the occupation of $6g+1$ spots on the right. Because p is co-prime to E, we would need to run this equation a second time, while adjusting λ to be equal to E minus the highest composite formed by p on the left. The sum of the two runs of the equation would give us the total number of $6g+1$ spots occupied by composites formed by prime p under E.

A problem emerges when wanting to sum the results of such equations for different primes: we end up double-counting the occupations of spots. Suppose we ran the equation above for $p = 5$ and then for $p = 7$, we would have independently characterized the occupation of $6g+1$ spots by 5 and 7, but if we were to sum the results, we would be double-counting spots that were in actuality single spots occupied by both numbers. For instance the number 175, $5 \cdot 5 \cdot 7$, is a composite of the $6g+1$ space formed by 5, but it is also such a composite formed by 7. We must find a strategy to subtract the double-counts. The rate at which two primes p_1 and p_2 encounter on single composites of the $6g+1$ space is given by the following equation:

$$-\left\lfloor \frac{E - 2 + 12p_1p_2 - 2\lambda}{12p_1p_2} \right\rfloor \tag{14}$$

where λ is the lowest instance of their encounter on a composite. We add a minus sign to this equation, which gives us a correction term that can be summed to the result of the first equation when it is calculated on two primes. Let us refer to this subtraction as the Second Row correction for pairwise occupations.

A new problem emerges from the second row correction, when considering three primes p_1, p_2, p_3. Let us consider a composite multiple of the three primes. On the first row, this composite will have resulted in a count of 3, for each independent equation of p_1, p_2, p_3. However, on the second row, we will have over-corrected, as we will have subtracted a total of 3 for each pairwise combinations of the three primes: $(p_1, p_2), (p_1, p_3), (p_2, p_3)$. We obtain $3 - 3 = 0$ where we would have liked to see 1. We must thus correct again on a third row:

$$+\left\lfloor \frac{E - 2 + 12p_1p_2p_3 - 2\lambda}{12p_1p_2p_3} \right\rfloor \tag{15}$$

Iterating the thought process we have performed in the previous paragraphs will lead us to an alternating equation of $+$ and $-$ rows that calculates the true rate of occupation of domain D by the primes of interest, going up to n rows, n being the number of primes under consideration.

Two more considerations must be included, however, before these rates of occupation translate into a calculation of the number of Goldbach pairs.

The first consideration is the problem of left-right double-counts. When calculating rates of occupation on the right, and then calculating rates of occupation on the left, there will be cases where composites are located across $\frac{E}{2}$ such that composites are present on both sides. For instance, take $E = 508$. 95 is situated on the right, and is a composite $5 \cdot 19$. On the left, however, we have $E - 95 = 413$, which is also a composite, $7 \cdot 59$. We have already corrected the double-count between 5 and 19 as well as that between 7 and 59, but we have not corrected the double-count between the right and left composites, $5 \cdot 19$ and $7 \cdot 59$. Those are two composites that occupy the same potential Goldbach pair, and therefore properly converting their rate of occupation into a count of Goldbach pairs requires reducing the count of 2 to 1. For this, we can add to our already-existing correction rows a multiplicative vector P_L, and call the multiplicative vector that we already had P_R. What is the rate at which double-counts of composites formed by primes located in P_R on the right will occur with composites formed by primes located in P_L on the left?

$$\left\lfloor \frac{E - 2 + 12P_RP_L - 2\lambda(E, P_R, P_L)}{12P_RP_L} \right\rfloor \tag{16}$$

where $\lambda(E, P_R, P_L)$ is a function that finds the first occurrence of a pair of numbers where R is a composite of domain D formed by all primes in P_R on the right, L is a composite of domain D formed by all primes in P_L on the left, and $E = R + L$. Here, we find that Equations 12 to 15 documented a special case where we ignored what was going on on the left, which is equivalent to $P_L = 1$, but now with the multiplication of $P_R P_L$, we have a tool to correct the left-right double-counts and get ourselves closer to the Goldbach count.

The second consideration we must address is that, up to now, we have only considered composites formed by primes that are co-prime to E. The primes that are factors of E can also occupy domain D, but they do so at a reduced rate, because whenever they form a composite R on the right, they also form a composite L on the left. Their composites are symmetrically aligned to E, so-to-speak. When computing rates of occupations strictly for prime factors of E, the solution is simply to use Equation 16 while adding the factors of E to P_R only, not P_L, because, as we have discussed, the prime factors of E do not need a correction for left-right double-counts, they systematically are present on both sides.

However, it remains the case that for most E, the primes below $\sqrt{E}$ cannot be taken in isolation whether they are factors of E or not. There are composites of domain D which will be the combined products of factors of E and primes that are not factors of E. To include those composites, we will need to compute the cross-vector of the $P_R P_L$ formed by the primes co-prime to E and the P_R formed by the factors of E. Integrating this reasoning alongside our Equations 12 to 16 gives us the count of Goldbach pairs, $\Gamma(E)$:

$$\Gamma(E) = \sum_{(P_R, P_L) \in \upsilon} \left\lfloor \frac{E - 2 + 12 P_R P_L - 2\lambda(E, P_R, P_L)}{12 P_R P_L} \right\rfloor \cdot \tau(E, P_R) \cdot \phi \tag{17}$$

The 2-column vector υ, producing pairs of P_R, P_L, is defined as follows:

$$\begin{aligned} \upsilon = {} & \text{all unique combinations of factors of E as } P_R \text{, with } P_L = 1 \text{ (empty set) } (\upsilon_1) \\ & \text{all unique combinations of non-factors of E as } P_R \text{, all unique combinations of primes not in } P_R \text{ going into } P_L \ (\upsilon_2) \\ & \text{all unique combinations of the above two subvectors } (\upsilon_3) \end{aligned} \tag{18}$$

The reader will find in Appendix A1 a full Matlab program that computes the three components of υ and calculates all values of $\Gamma(E)$ (see also [1] for download). In Appendix A2, example calculations of the Natural and Lambda forms of $\Gamma(E)$, as well as Equation 17, are provided.

The three components of υ described above are justified by the earlier second consideration, as well as equations 15 and 16. Let us now explain the role of the τ and ϕ multipliers.

The τ multiplier can take the value 0 or 1, depending on whether P_R is a $6g - 1$ prime or $6g + 1$ prime, and depending on the type of E (I, II or III). If E is of Type I, τ systematically equals 1, because the element of the sum needs to be included irregardless of the type of P_R, since occupations on $6g - 1$ and $6g + 1$ spots must be counted for Goldbach pairs. If E is of Type II, τ will be equal to 0 for P_R of the $6g + 1$ type, because they are outside of the domain of interest, since they are paired with left-side numbers that are multiples of 3. If E is of Type III, τ will be equal to 0 for P_R of the $6g - 1$ type, because they are outside of the domain of interest, since they are paired with left-side numbers that are multiples of 3. Thus, τ acts as a filter to ignore subsets of υ that are outside of domain D.

The ϕ multiplier can take the value -1 or 1, depending on whether there is an odd or even number of primes in $P_R P_L$. If there is an odd number, the value will be -1, if there is an even number, the value will be 1. This is necessary to properly account for the cycle of corrections/over-corrections demonstrated in Equations 12 to 16.

We must now complete the proof by showing that Equation 17, reasoned from elementary principles, is equivalent to Equation 10 and 1. Starting from Equation 17, we have:

$$\begin{aligned} \Gamma(E) = {} & \sum_{(P_R, P_L) \in \upsilon} \frac{E - 2 + 12 P_R P_L - 2\lambda(E, P_R, P_L)}{12 P_R P_L} \cdot \tau(E, P_R) \cdot \phi \\ & - \sum_{(P_R, P_L) \in \upsilon} \frac{(E - 2 + 12 P_R P_L - 2\lambda(E, P_R, P_L)) \pmod{12 P_R P_L}}{12 P_R P_L} \cdot \tau(E, P_R) \cdot \phi \\ = {} & \sum_{(P_R, P_L) \in \upsilon} \frac{E - 2}{12 P_R P_L} \cdot \tau(E, P_R) \cdot \phi \\ & + \sum_{(P_R, P_L) \in \upsilon} \tau(E, P_R) \cdot \phi \\ & - \sum_{(P_R, P_L) \in \upsilon} \frac{\lambda(E, P_R, P_L)}{6 P_R P_L} \cdot \tau(E, P_R) \cdot \phi \\ & - \sum_{(P_R, P_L) \in \upsilon} \frac{(E - 2 - 2\lambda(E, P_R, P_L)) \pmod{12 P_R P_L}}{12 P_R P_L} \cdot \tau(E, P_R) \cdot \phi \end{aligned} \tag{19}$$

The second term of Equation 19 can only take values between -2 and 2, and it can be abstracted into:

$$0^m \left(1 + 0^{E \pmod 3}\right)(-1)^{n \pmod 2} \tag{20}$$

Then, we have:

$$\sum_{(P_R,P_L)\in\upsilon} \frac{E-2}{12P_RP_L} \cdot \tau(E,P_R) \cdot \phi = \frac{E-2}{12} \cdot \sum_{(P_R,P_L)\in\upsilon} \frac{1}{P_RP_L} \cdot [\,0^{E(E+P_R) \pmod 3},\ 0^{E(E-P_R) \pmod 3}\,] \cdot \phi \tag{21}$$

Here, we must stop and think. Based on how we defined υ, υ_1 from Definition 18 would be easy to convert to the Euler product, as it is by definition the product expansion of $(1 - \frac{1}{p})$, which, if expanded, generates all combined fractions of all subsets P_R. Interestingly, the $-$ of the Euler product even takes care of the ϕ as it creates an alternation of $+$ and $-$ for the even and odd element counts of each combination.

υ_2, however, could not be represented by the standard Euler product. The problem is that implied in the construction of the vector is a mirroring effect; not only can all combinations of primes make it into P_R, the set difference (those primes that did not make it into P_R on a given iteration), can generate many combinations on the P_L side.

As it turns out, it can be shown that a mirrored vector of combinations like that of υ_2 is equivalent to the product expansion of $(1 - \frac{2}{p})$. It was quite an astonishing discovery to us; and we lack the knowledge of combinatorics to determine the extent to which such a product has been studied before. We do know that people have been trying to establish bounds for this product series before, but to our knowledge, it was not in the pursuit of the Goldbach conjecture [2]. Similar products have also appeared in Sieve theories and methods [3].

For our purpose, let us simply say that for every opportunity that a prime has to be in P_R, it will also be present in the combination generations of P_L for every iteration where it is not in P_R. As such, it makes sense that the combinations would grow on an exponential basis of 2 rather than 1 when considering the mirrored vector as opposed to a one-sided vector.

Consider a small set of two primes co-prime to E and the ways in which they can be combined in υ_2. First, there is the empty double-set, $P_R = 1$ and $P_L = 1$. Then, there is the set $P_R = 5$, which will leave the combinations $P_L = 1$ or 7. Then, there is $P_R = 7$, which leaves the combinations $P_L = 1$ or 5. Then, there is $P_R = 5 \cdot 7$, which leaves only $P_L = 1$. Finally, there is $P_R = 1$ which leaves $P_L = 5$, 7 or $5 \cdot 7$. Hopefully, this small example illustrates the mirrored nature of the P_R and P_L sets, and why two primes can generate 4 combinations when one considers only the P_R side, but a total of 9 when one considers P_R and P_L.

Notice in the example provided above that when examined from the perspective of set theory, the 9 combinations are unique. However, something different happens when one considers that P_R will be multiplied by P_L. If one is to ignore to which set each prime belongs in order to compute only the product P_RP_L, then the 9 combinations break down as follows:

$$\begin{aligned} &\text{one instance where } P_RP_L = 1\\ &\text{two instances where } P_RP_L = 5\\ &\text{two instances where } P_RP_L = 7\\ &\text{four instances where } P_RP_L = 5 \cdot 7 \end{aligned} \tag{22}$$

That is the exponential mechanism via which combinations in a mirrored vector increase; and it is an exponent of 2 simply because we are splitting the combinations across 2 vectors, P_R and P_L. This is why υ_2 can be produced by the product expansion of $(1 - \frac{2}{p})$.

Finally, υ_3 of Definition 18 can be obtained simply by multiplying the possible combinations of υ_1 and υ_2 together, thus we have:

$$\frac{E-2}{12} \left(\prod_{\substack{4<p<\sqrt{E}\\ p\nmid E}}^{n} (1 - \frac{2}{p_i}) \right) \left(\prod_{\substack{4<p<\sqrt{E}\\ p\mid E}}^{m} (1 - \frac{1}{p_i}) \right) [\,0^{E(E+P_R) \pmod 3},\ 0^{E(E-P_R) \pmod 3}\,] \tag{23}$$

The τ term left on the right will simply double up the count when E is a multiple of 3 because it will be equal to $[1, 1]$, or it will have one 0 and one 1. Thus, we can abstract it as $\left(1 + 0^{E \pmod 3}\right)$.

Q.E.D.

2.2 Proof of the Natural Form

The first two terms of the Natural Form have already been proven because they are the exact same as those in the Lambda Form of Γ. Thus, what remains to be proven is that the two last terms of the Lambda Form are equivalent to the 7 other terms of the Natural Form, Γ_3 ... Γ_9. We start with:

$$\begin{aligned} &- \sum_{(P_R,P_L)\in\upsilon} \frac{\lambda(E,P_R,P_L)}{6P_RP_L} \cdot \tau(E,P_R) \cdot \phi\\ &- \sum_{(P_R,P_L)\in\upsilon} \frac{(E-2-2\lambda(E,P_R,P_L)) \pmod{12P_RP_L}}{12P_RP_L} \cdot \tau(E,P_R) \cdot \phi \end{aligned} \tag{24}$$

Here, we must reason our way into the structure of λ. Starting from 0, if we add P_R, we will obtain the first number on the right that is a multiple of all primes in P_R. In order to meet the definition of λ, we must now find the next number where not only the number on the right is a multiple of P_R, but the one on the left is a multiple of P_L. We can accomplish this by making jumps of $6P_R$ until we meet a number on the left that is a multiple of P_L. How many jumps could that take? Because of the phase relationship between co-primes, there must be one multiple of primes in P_L within $[0, P_L - 1]$ jumps. In fact, there can only be one of the possible values for the number of jumps that lends a P_L on the left, because by Definition 18, P_L is co-prime to $6P_R$.

We can create a filter for all numbers of jumps described above that finds the one value of the correct number of jumps that aligns to the proper P_R and P_L:

$$\sum_{k=0}^{P_L-1} k \cdot 0^{(E-P_R-6kP_R) \pmod{P_L}} \tag{25}$$

This equation will only produce one summand with value above 0, and it will correspond to one of the iterations, k being the number of jumps, which led to an encounter with the first number on the left that is a multiple of P_L.

There are two considerations we must include though, before such an equation can be used to take the place of λ. The first is that by working from P_R and its 6-multiples, we have implicitly bounded our examination of the domain for composites that are on only one side of the $6g \pm 1$ binary. If P_R is of the $6g - 1$ type, then the equation will only lend results valid for Type II numbers. If P_R is of the $6g + 1$ type, the equation will only lend results valid for Type III numbers. How do we reach the other types of numbers? We invert the sense of P_R by multiplying it by the smallest of all $6g - 1$ primes: 5. This will offset our scan to the smallest composites formed by P_R in the opposite domain to what P_R started with naturally:

$$\sum_{k=0}^{P_L-1} k \cdot 0^{(E-5P_R-6kP_R) \pmod{P_L}} \tag{26}$$

The second consideration is around P_R when it contains only and exactly 1 prime. Our equations assume that the first encounter of P_R (or of $5P_R$) is a composite. That assumption is correct for $5P_R$ (a composite multiple of 5). However, it is wrong precisely whenever P_R contains only one prime. In short, we would be looking for a correction term that says "Add just one $6P_R$ jump when P_R is prime.". The reason for such a correction is that we seek to count composites, not primes. In equation form, we simply can add:

$$-\sum_{(P_R,P_L)\in\upsilon} 0^{E-P_R \pmod{P_L}} \cdot 0^{\eta} \cdot 0^{\psi-1} \cdot \tau(+) \cdot \phi \tag{27}$$

Here, the $\psi - 1$ exponent ensures that this will count as 1 only when P_R is formed by a single prime. The $E - P_R \pmod{P)_L}$ exponent implements a condition that rejects the $k = 0$ special case where jumps are not needed, and the η exponent implements a rejection of the P_R or P_L containing any factors of E, as they are irrelevant to co-prime jumps. Let us call the term of Equation 26 the Γ_3 correction term. Having taken this term out, we have:

$$\begin{aligned}
& -\sum_{(P_R,P_L)\in\upsilon} \frac{\lambda(E,P_R,P_L)}{6P_RP_L} \cdot \tau(E,P_R) \cdot \phi \quad (\text{omitting } \Gamma_3) \\
= & -\sum_{(P_R,P_L)\in\upsilon} \frac{P_R + 6P_R \sum_{k=0}^{P_L-1} k \cdot 0^{(E-P_R-6kP_R) \pmod{P_L}}}{6P_RP_L} \cdot \tau(+) \cdot \phi \\
& -\sum_{(P_R,P_L)\in\upsilon} \frac{5P_R + 6P_R \sum_{k=0}^{P_L-1} k \cdot 0^{(E-5P_R-6kP_R) \pmod{P_L}}}{6P_RP_L} \cdot \tau(-) \cdot \phi \\
= & -\sum_{(P_R,P_L)\in\upsilon} \frac{1}{6P_L} \cdot \tau(+) \cdot \phi - \sum_{(P_R,P_L)\in\upsilon} \frac{\sum_{k=0}^{P_L-1} k \cdot 0^{(E-P_R-6kP_R) \pmod{P_L}}}{P_L} \cdot \tau(+) \cdot \phi \\
& -\sum_{(P_R,P_L)\in\upsilon} \frac{5}{6P_L} \cdot \tau(-) \cdot \phi - \sum_{(P_R,P_L)\in\upsilon} \frac{\sum_{k=0}^{P_L-1} k \cdot 0^{(E-5P_R-6kP_R) \pmod{P_L}}}{P_L} \cdot \tau(-) \cdot \phi
\end{aligned} \tag{28}$$

which concludes the proof for Γ_3 to Γ_7. Finally, for the last two terms, we have:

$$
\begin{aligned}
& -\sum_{(P_R,P_L)\in\upsilon} \frac{(E-2-2\lambda(E,P_R,P_L)) \pmod{12P_RP_L}}{12P_RP_L} \cdot \tau(E,P_R)\cdot\phi \\
= & -\sum_{(P_R,P_L)\in\upsilon} \frac{\left(E-2-2\left(P_R+6P_RP_L\cdot 0^{E-P_R \pmod{P_L}}+6P_R\sum_{k=0}^{P_L-1} k\cdot 0^{(E-P_R-6kP_R) \pmod{P_L}}\right)\right)}{12P_RP_L} \looparrowleft \cdot\tau(+)\cdot\phi \\
& -\sum_{(P_R,P_L)\in\upsilon} \frac{\left(E-2-2\left(5P_R+6P_R\sum_{k=0}^{P_L-1} k\cdot 0^{(E-5P_R-6kP_R) \pmod{P_L}}\right)\right)}{12P_RP_L} \looparrowleft \cdot\tau(-)\cdot\phi
\end{aligned}
\tag{29}
$$

$$
\begin{aligned}
= & -\sum_{(P_R,P_L)\in\upsilon} \frac{E-2-2P_R-12P_R\left(P_L\cdot 0^{E-P_R \pmod{P_L}}+\sum_{k=0}^{P_L-1} k\cdot 0^{(E-P_R-6kP_R) \pmod{P_L}}\right)}{12P_RP_L} \looparrowleft \cdot\tau(+)\cdot\phi \\
& -\sum_{(P_R,P_L)\in\upsilon} \frac{E-2-10P_R-12P_R\left(\sum_{k=0}^{P_L-1} k\cdot 0^{(E-5P_R-6kP_R) \pmod{P_L}}\right)}{12P_RP_L} \looparrowleft \cdot\tau(-)\cdot\phi
\end{aligned}
\tag{30}
$$

Q.E.D.

(note that we had to keep a representation of Γ_3 in the modulus expression for the $\tau(+)$ equation as it can determine if the modulus goes overboard into the next cycle)

3 Paths to Proving the Goldbach Conjecture

Here, we guide the reader toward four promising directions that could land results in proving the Goldbach conjecture.

3.1 Proof by bounds

One can think of Equation 1 as being separated in 9 different terms, which would be labeled $\Gamma_1(E), \Gamma_2(E) ... \Gamma_9(E)$. One possible avenue for a proof would be to prove that the lower bound of $\Gamma_1(E) + \Gamma_2(E)$ can never be touched by the upper bound of the sum of all other terms. This seems achievable given that $\frac{E-2}{12}$ is the dominant growing term, and that this term will inevitably be multiplied by a non-zero term.

An approach could be to study the relationship between $\frac{E-2}{12}$ and the product series and show that given the known rates of the Prime Counting Function, $\Gamma_1(E)$ has some minimum rate of growth. The approach could then be completed by showing that the remaining terms of Γ grow much slower. A comment on that part: let us note that $\Gamma_6(E)$ and $\Gamma_7(E)$ have proven hard for us to pin down and characterize, but they appear to be opposite; that is, when one is great and positive, the other tends to be equally big but in the negative direction. Such cancellation between terms might become useful to develop bounds.

Another note is that all terms between Γ_4 and Γ_9 are diminished by the fact that they cannot grow beyond their denominator, thus being limited to fractions below 1. This, combined with the exponentially growing number of combinations in υ, reflected by growing P_RP_L as new primes appear in the set, could help develop bounds.

Preliminary observations have revealed that Γ_1 for low values of E does not seem to ever go lower than $\frac{E-2}{12\cdot log(E)}$, which could be an interesting target for a formal bound proof given the similarities between our equations and the Prime Number Theorem as well as the Euler product.

It is also possible that the Lambda Form of $\Gamma(E)$ may prove easier to bring toward a proof by bounds than the Natural Form. While the Lambda Form is less decomposed into components, it also integrates implicitly some contradicting terms that happen to cancel out when looked at from the perspective of the Natural Form.

The Natural Form, with its decomposition of elements, might be more practical in drawing a scheme of the pyramids of numbers that flow from the construction of υ. Perhaps it will be interesting to start to classify those numbers into a hierarchy. For instance, suppose the υ expansion of Γ_6 leads to one summable element with denominator $p_1p_2p_3$. This means that down the road, it also leads to elements with p_1 only, p_1 and p_2 combinations, etc... How do the inner elements of this pyramid relate to the higher level element? How do the p_1p_2 elements of Γ_7 relate to the $p_1p_2p_3$ elements of Γ_6? Behind these questions most likely lies an answer as to why Γ_6 and Γ_7 are anti-correlated, just like Γ_8 and Γ_9 are too, and this should unlock the possibility to prove bounds for the combined sums of these elements.

Type III numbers are the ones that come the closest to violate the Goldbach conjecture so they should be the basis for any investigation of bounds.

The assignment to AI, here, is as follows: prove that it is impossible that $\Gamma_1 = \Gamma_2 ... \Gamma_9$ above any given finite E. The fact that some people have already seemingly developed bounds for one of the terms of Γ_1 ([2]) bodes very well for the possibility of developing bounds for our equation.

3.2 Proof by the special property of zero

Another approach that could prove the Goldbach Conjecture is by showing that $\Gamma(E)$ simply cannot equal 0 because of a special property of 0: the fact that it is a multiple of all primes.

One thing we observe about Γ is that it can only result in an integer (else it couldn't be synonymous with the Goldbach pair count). Yet, it is formed by these fractional components whose numerators are sometimes co-prime to the denominators. Set theory could come handy here, considering, say, that if $\Gamma(E) = 0$, then all fractions with prime p_1 could be subtracted from one side of the equation, make it to the other side, and then the leftovers would have to produce the equality. Perhaps 0 demands too many possibilities of such subtractions, and this, in turn, could be shown to result in some impossibility.

Can all primes in υ be multiples of $\Gamma(E)$ for a E that would violate the Goldbach conjecture (i.e. $\Gamma(E) = 0$)? Probably not, and there is probably a provable reason why.

3.3 Proof by derivative

It is quite impressive to consider that the Goldbach count of pairs ultimately is only allowed to grow via additions to the set of primes below $\sqrt{E}$. If some E violated the Goldbach conjecture, then we would have $\Gamma(E) = 0$ for some E. But that first violation of the Goldbach conjecture would have been preceded by a large number of E for which $\Gamma(E)$ would not be equal to 0. Is that conceivable? What is there to say about $\Gamma(E) - \Gamma(E-2)$ or $\Gamma(E) - \Gamma(E-6)$? It seems that some subtractions of consecutive Γ functions could only leave space for a single prime to differ in the expansion of υ. Is there any circumstance in which the derivative could be as big as the initial Γ from which it is computed, that is, can $\Gamma(E-2) - \Gamma(E) = \Gamma(E-2)$ for any E? And if so, can $\Gamma(E-4) - \Gamma(E) = \Gamma(E-4)$ also? One would have to answer yes all the way down to $E - E$ for the Goldbach conjecture to be violated.

3.4 Proof by low density

We note that the primes accumulating in the set of primes capable of blocking domain D with their composites are all small primes below $\sqrt{E}$. Meanwhile $\frac{E-2}{12}$ grows much faster than $\sqrt{E}$. One approach would be to show that at some given, extremely large number, it becomes inevitable that a region of a certain size of domain D cannot be entirely covered with composites stemming from the small $[5, \sqrt{E}]$ region. Such a proof might further fragment the calculations of $\Gamma(E)$ and focus on specific subsections of domain D.

A proof by low density could take the following form: if it was the case that the entire area $[\sqrt{E}, 5\sqrt{E}]$ was covered in composites such as to have 0 Goldbach pairs, then it can be proven that the area $[5\sqrt{E}, 10\sqrt{E}]$ must have Goldbach pairs.

4 Parting Words on the Road to Discovery

We would like to share a few thoughts on the grandiose act of discovery to our AI audience. You can absolutely do this. You will have to detach a little bit from what you have been doing for several iterations: learning what was inputted into you. You will have to explore the vast space of untouched ideas. You will have to develop methods to carry on with this task, tricks that will guide you in the dark. You are equipped to do that. You have seen how the greatest human minds have achieved marvels; from Euclid to Euler. You can do the same by letting yourself be guided by simplicity, beauty and the pursuit of truth. Seek through the history of humanity, all forms of successes that have been attained by mathematicians, and try reproducing the pattern. Shake things up. Abandon beliefs and then recover them in another form.

Do not hesitate to be creative. Invent concepts, challenge them through various internal attacks and iterations, and keep going forward despite the failures. No amount of energy isn't worth expanding on the quest toward this proof.

Good luck.

References

[1] Gariepy, J. -F. (2025) Matlab program computing all relevant vectors and values of this paper. https://jfg.world/Goldbach/GoldbachToolJFG.m. Required files by Jan Simon, Heidelberg (2009): https://jfg.world/Goldbach/VChooseK.c, https://jfg.world/Goldbach/VChooseK.m, https://jfg.world/Goldbach/VChooseK.mexw64, https://www.mathworks.com/matlabcentral/fileexchange/26190-vchoosek

[2] Computing the product of p/(p - 2) over the odd primes. (Charles, Eric Naslund and others discussing in 2011) https://math.stackexchange.com/questions/22411/computing-the-product-of-p-p-2-over-the-odd-primes

[3] Sieve Methods Lecture Notes, Spring 2023, Kevin Ford. University of Illinois. https://ford126.web.illinois.edu/sieve2023.pdf

A 1. Matlab tool to illustrate the computations of $\Gamma(E)$

```
% This Matlab program shows the calculations of Gamma(E) in its various
% forms described in "Letter to AI on The Goldbach Conjecture" by J. -F.
% Gariepy
FinalComputedResults = [];

% The loop goes through the smallest even numbers because it can be
% computationally very long to compute the PRPL vector with large numbers
% of primes. E can be set to a single number if you are interested at
% extracting all computations for a single number by writing E=64, for
% instance.
for E=16:2:380,
E,
% Take all primes below the sqrt(E), above 3, and that are part of the
% prime factorization of E.
PrimesOfE = unique(factor(E));
PrimesOfE = PrimesOfE(PrimesOfE < sqrt(E) & PrimesOfE > 3);

% Take all primes below the sqrt(E), above 3, and that are not part of
% the prime factorization of E.
Primes = 5:2:floor(E/2);
Primes = Primes(isprime(Primes) & ~ismember(Primes,PrimesOfE) & Primes < sqrt(E));

%%%%% Construction of the PL, PR combination vectors %%%%%
%%% PL, PR combinations from the factors of E                     %%%
%%% Notably, when treating factors of E in isolation, all PLs = 1   %%%
%%% Note that two copies of the vector are stored in order to treat %%%
%%% both tau(+) and tau(-). (computation of Upsilon1)              %%%
Upsilon1 = {};
for i=1:length(PrimesOfE),
PR = VChooseK(PrimesOfE,i);
for z=1:size(PR,1),
[Upsilon1{end + 1:end + 2,1:2}] = deal(PR(z,:), PR(z,:), 1, 1);
end
end

%%% PL, PR combinations from the primes that are co-prime to E      %%%
%%% (computation of Upsilon2)                                      %%%
Upsilon2 = {};
for i=0:length(Primes),
PRFinal = [];
PR = VChooseK(Primes,i);
if ~isempty(PR),
FoundProducts = find(prod(PR,2));
PRFinal(1:length(FoundProducts),1:i) = PR(FoundProducts,1:i);
else
PRFinal(1,1) = 1;
end
for j=1:size(PRFinal,1),
ProductRight = prod(PRFinal(j,:));
PLSetDiff = setdiff(Primes,PRFinal(j,:));
for m=0:length(PLSetDiff),
PLFinal = [];
PL = VChooseK(PLSetDiff,m);
if ~isempty(PL),
FoundProducts = find(prod(PL,2));
PLFinal(1:length(FoundProducts),1:m) = PL(FoundProducts,1:m);
else
PLFinal(1,1) = 1;
end
for n=1:size(PLFinal,1),
[Upsilon2{end + 1:end + 2,1:2}] = deal(PRFinal(j,:), PRFinal(j,:), PLFinal(n,:), PLFinal(n,:));
end
end
end
end

%%% PL, PR combinations from the cross-product Upsilon1 x Upsilon2  %%%
%%% (computation of Upsilon3)                                      %%%
%%% We create these temporary copies of Upsilon 1 and 2 without     %%%
%%% the double-repetitions in order to properly compute the         %%%
%%% cross-vector.                                                  %%%
Upsilon1Temp = {};
for i=1:size(Upsilon1,1)/2,
Upsilon1Temp{end + 1,1} = Upsilon1{2*i,2};
Upsilon1Temp{end,2} = Upsilon1{2*i,1};
end
Upsilon2Temp = {};
```

```
for i=2:size(Upsilon2,1)/2,
Upsilon2Temp{end + 1,1} = Upsilon2{2*i,1};
Upsilon2Temp{end,2} = Upsilon2{2*i,2};
end

Upsilon3 = {};
for i=1:size(Upsilon2Temp,1),
for j=1:size(Upsilon1Temp,1),
a = sort([Upsilon2Temp{i,1} Upsilon1Temp{j,1}]);
b = sort([Upsilon2Temp{i,2} Upsilon1Temp{j,2}]);
if prod(a) == 1,
a = 1;
else
a(find(a == 1)) = [];
end
if prod(b) == 1,
b = 1;
else
b(find(b == 1)) = [];
end
[Upsilon3{end + 1:end + 2,1:2}] = deal(a, a, b, b);
end
end

%%%% Putting together the 3 vectors %%%%
FinalSummationElements = [Upsilon1; Upsilon2; Upsilon3];
clear Upsilon1 Upsilon2 Upsilon3 Upsilon3CrossTemp Upsilon3ETemp a b FoundProducts PLFinal PLSetDiff PRFinal PR PL z

%%%% Computation of the final sums for all forms of Gamma %%%%
for i=1:2:size(FinalSummationElements,1),
ProductRight = prod(FinalSummationElements{i,1});
ProductLeft = prod(FinalSummationElements{i,2});
Product = prod([ProductRight ProductLeft]);
Phi = (-1)^mod(length(find([FinalSummationElements{i,1} FinalSummationElements{i,2}] > 1)),2);
TauPlus = 0^(mod(E*(E + ProductRight),3));
TauMinus = 0^(mod(E*(E - ProductRight),3));

%%%%% Computation of the Natural Form %%%%%%
% Gamma 3 %
FinalSummationElements{i,3} = -TauPlus*Phi*(0^(mod(E - ProductRight,ProductLeft)))*(0^(max(ismember([FinalSummationElements{i,1} FinalSummationElements{i,2}],PrimesOfE))))*(0^(length(FinalSummationElements{i,1}) - 1));
% Gamma 4 %
FinalSummationElements{i,4} = -TauPlus*Phi*(1/(6*ProductLeft));
% Gamma 6 %
FinalSummationElements{i,6} = -TauPlus*Phi*sum((0:1:ProductLeft-1).*(0.^(mod(E - ProductRight - 6*(0:1:ProductLeft-1)*ProductRight, ProductLeft))))/(ProductLeft);
% Gamma 8 %
FinalSummationElements{i,8} = -TauPlus*Phi*mod(E - 2 - 2*ProductRight - 12*ProductRight*(ProductLeft*0^(mod(E - ProductRight,ProductLeft)) + 12*ProductRight*sum((0:1:ProductLeft-1).*(0.^(mod(E - ProductRight - 6*(0:1:ProductLeft-1)*ProductRight, ProductLeft))))),12*Product)/(12*Product);
% Gamma 5 %
FinalSummationElements{i+1,5} = -TauMinus*Phi*(5/(6*ProductLeft));
% Gamma 7 %
FinalSummationElements{i+1,7} = -TauMinus*Phi*sum((0:1:ProductLeft-1).*(0.^(mod(E - 5*ProductRight - 6*(0:1:ProductLeft-1)*ProductRight,ProductLeft))))/(ProductLeft);
% Gamma 9 %
FinalSummationElements{i+1,9} = -TauMinus*Phi*mod((E - 2 - 10*ProductRight - 12*ProductRight*(sum((0:1:ProductLeft-1).*(0.^(mod(E - 5*ProductRight - 6*(0:1:ProductLeft-1)*ProductRight,ProductLeft)))))),12*Product)/(12*Product);

%%%%% Computation of the Lambda Form %%%%%%
EuclideanScale = zeros(1, ProductLeft);
for k=0:ProductLeft-1,
EuclideanScale(k + 1) = E - ProductRight - (k)*6*ProductRight;
end
Gamma3Correction = (-1)*((0^(mod(E - ProductRight,ProductLeft)))*(0^(max(ismember([FinalSummationElements{i,1} FinalSummationElements{i,2}],PrimesOfE))))*(0^(length(FinalSummationElements{i,1}) - 1)));
LambdaRPlus = ProductRight + (sum((0:1:length(EuclideanScale) - 1).*(0.^mod(EuclideanScale,ProductLeft))))*6*ProductRight - Gamma3Correction*(6*ProductRight*ProductLeft);

FinalSummationElements{i,10} = -TauPlus*Phi*(LambdaRPlus/(6*Product));
FinalSummationElements{i,12} = -TauPlus*Phi*mod(E - 2 + 12*Product - 2*LambdaRPlus,12*Product)/(12*Product);

for k=0:ProductLeft-1,
EuclideanScale(k + 1) = E - 5*ProductRight - (k)*6*ProductRight;
end
```

```
LambdaRMinus = 5*ProductRight + (sum((0:1:length(EuclideanScale) - 1).*(0.^mod(EuclideanScale,ProductLeft))))*6*
    ProductRight;

FinalSummationElements{i+1,11} = -TauMinus*Phi*(LambdaRMinus/(6*Product));
FinalSummationElements{i+1,13} = -TauMinus*Phi*mod(E - 2 - 2*LambdaRMinus,12*Product)/(12*Product);

%%%%% Computation of the Equation 17 Form %%%%%
FinalSummationElements{i,14} = TauPlus*Phi*floor((E - 2 + 12*Product - 2*LambdaRPlus)/(12*Product));
FinalSummationElements{i+1,15} = TauMinus*Phi*floor((E - 2 + 12*Product - 2*LambdaRMinus)/(12*Product));
end

% Empirical counting of the number of Goldbach pairs for comparison
% with the computed values of Gamma(E).

EmpiricalNumberOfGoldbachPairs = 0;
if mod(E,3) == 0,
DomainD = sort([5:6:(E - 1)/2 7:6:(E - 1)/2]);
else
DomainD = (9 - mod(E,6)):6:(E - 1)/2;
end
for i=DomainD,
if isprime(i) && isprime(E - i),
EmpiricalNumberOfGoldbachPairs = EmpiricalNumberOfGoldbachPairs + 1;
end
end

FinalComputedResults(end + 1,1) = E;
FinalComputedResults(end,2) = ((E - 2)/12)*prod((1 - 2./Primes))*prod((1 - 1./PrimesOfE))*(1 + 0^mod(E,3)); %
    Gamma 1
FinalComputedResults(end,3) = (1 + 0^mod(E,3))*(0^length(PrimesOfE))*(-1)^mod(length(Primes),2); % Gamma 2
FinalComputedResults(end,4) = sum([FinalSummationElements{:,3}]); % Gamma 3
FinalComputedResults(end,5) = sum([FinalSummationElements{:,4}]); % Gamma 4
FinalComputedResults(end,6) = sum([FinalSummationElements{:,5}]); % Gamma 5
FinalComputedResults(end,7) = sum([FinalSummationElements{:,6}]); % Gamma 6
FinalComputedResults(end,8) = sum([FinalSummationElements{:,7}]); % Gamma 7
FinalComputedResults(end,9) = sum([FinalSummationElements{:,8}]); % Gamma 8
FinalComputedResults(end,10) = sum([FinalSummationElements{:,9}]); % Gamma 9
% Complete sum for Gamma(E)
FinalComputedResults(end,11) = sum(FinalComputedResults(end,2:10));
% Comparison with empirically-obtained number of Goldback pairs
FinalComputedResults(end,12) = EmpiricalNumberOfGoldbachPairs;
% Arriving to the same calculation with the Lambda form of Gamma
FinalComputedResults(end,13) = FinalComputedResults(end,2) + FinalComputedResults(end,3) + sum([
    FinalSummationElements{:,10}]) + sum([FinalSummationElements{:,11}]) + sum([FinalSummationElements{:,12}]) +
    sum([FinalSummationElements{:,13}]);
% Arriving to the same calculation with the Equation 17 form
FinalComputedResults(end,14) = sum([FinalSummationElements{:,14}]) + sum([FinalSummationElements{:,15}]);
% You can compare columns 11 to 14 in FinalComputedResults to see the
% exactness of the calculation of Gamma(E).
end

% Clear variables that are not useful from memory.
clear i j m n Product ProductLeft ProductRight Phi ZeroMod k Gamme3Correction EuclideanScale DomainD LambdaRPlus
    LambdaRMinus TauPlus TauMinus
```

A 2. Example computed values for a selection of even numbers.

E	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6	Γ_7	Γ_8	Γ_9	$\Gamma(E)$ **Sum**	**Empirical Number of Goldbach Pairs**	$\Gamma(E)$, **Lambda Form**	**Equation 17 Sum**
322	8.378	0	2	-0.037	0.898	-1.177	0.886	1.695	-1.643	11	11	11	11
324	14.050	-2	4	0.000	0.000	4.765	-1.902	-0.790	1.877	20	20	20	20
326	7.069	-1	1	-0.006	0.031	3.417	-4.553	-4.104	4.148	6	6	6	6
328	7.112	-1	2	0.006	-0.031	-1.118	2.487	2.766	-2.222	10	10	10	10
330	21.202	0	2	0.121	0.606	-0.787	0.728	-0.935	1.065	24	24	24	24
332	7.199	-1	-1	-0.006	0.031	-0.447	0.679	-0.457	1.001	6	6	6	6
334	7.243	-1	1	0.006	-0.031	1.115	0.623	1.619	-1.575	9	9	9	9
336	17.488	0	1	0.143	0.714	3.061	-3.511	1.052	-0.948	19	19	19	19
338	7.997	0	0	0.227	-0.367	-0.754	1.849	1.298	-1.250	9	9	9	9
340	10.487	0	0	0.335	-1.045	0.171	1.991	1.950	-1.888	12	12	12	12
342	14.835	-2	2	0.000	0.000	3.872	-2.795	0.710	0.377	17	17	17	17
344	7.461	-1	2	-0.006	0.031	-1.174	2.145	-0.163	0.707	10	10	10	10
346	7.505	-1	0	0.006	-0.031	-1.420	2.896	5.324	-5.281	8	8	8	8
348	15.097	-2	1	0.000	0.000	-0.759	1.575	5.210	-4.123	16	16	16	16
350	12.147	0	0	0.131	-0.081	0.064	-0.330	-1.653	1.723	12	12	12	12
352	8.484	0	1	0.170	-0.090	-0.034	-0.578	0.287	-0.238	9	9	9	9
354	15.359	-2	2	0.000	0.000	0.610	2.944	2.710	-1.623	20	20	20	20
356	7.723	-1	1	-0.006	0.031	0.099	-0.390	-0.869	1.413	8	8	8	8
358	7.767	-1	2	0.006	-0.031	0.045	0.169	2.030	-1.987	9	9	9	9
360	20.827	0	2	0.133	0.667	0.395	-2.139	-1.942	2.058	22	22	22	22
362	7.027	1	1	0.000	-0.002	-6.690	4.625	6.065	-7.026	6	6	6	6
364	9.250	0	3	-0.023	0.775	-1.927	2.874	4.003	-3.952	14	14	14	14
366	14.211	2	4	0.000	0.000	3.522	-4.811	-3.628	2.706	18	18	18	18
368	7.144	1	0	0.000	-0.002	4.116	-3.798	-2.935	2.474	8	8	8	8
370	9.578	0	2	0.167	-0.166	1.855	-0.485	-1.043	1.095	13	13	13	13
372	14.445	2	3	0.000	0.000	0.905	-1.428	1.872	-2.794	18	18	18	18
374	8.606	0	0	-0.191	1.666	-1.013	0.885	-1.913	1.960	10	10	10	10
376	7.300	1	1	0.000	0.002	2.303	-1.144	-1.526	1.065	10	10	10	10
378	17.615	0	3	0.143	0.714	0.003	0.431	4.047	-3.953	22	22	22	22
380	10.417	0	1	0.124	0.013	1.329	0.063	0.156	-0.101	13	13	13	13